Analysis of Target Detection Probability and Calculating the Required Number of Sensors

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***Abstract*— The main functionality of a surveillance wireless sensor network is to detect unauthorized traversals in a field. Prior to deployment, the adequate number of sensors can be determined based on simulations of the sensing coverage. In this paper, we propose an analytical deployment quality measure that is the probability of detecting a randomly positioned target by a set of binary sensors. Using this formulation, the required number of sensors to provide a threshold probability can be obtained. The sensing- and communication neighboring degrees of a sensor can also be calculated that can be used as a design criteria. The model is verified with simulations, and the outcomes closely match the analytical results.**

I. Introduction

Assume that a set of sensors are deployed randomly to a region to detect unauthorized traversals. What is the quality of the deployment? Is the deployed number of sensors adequate to provide the required quality in terms of breach detection? In prior work [1], we proposed experimental methods based on different deployment quality measures. In this paper, we propose an analytical method to determine the required number of sensors based on only the sensing coverage. The probability of detecting a randomly positioned target by a set of binary sensors is formulated. Using this formulation, it is possible to determine the required number of sensors to provide a required detection probability. Also, the expected number of sensors that cover any randomly chosen point in the field can be determined.

We define three problems which are related to each other. All of the problems are based on the following assumptions: The field is rectangular and $D_1 \times D_2$ m^2. The positions of the sensors are uniformly random. The x and y coordinates are independent. N identical sensors are deployed. Binary detectors with sensing range d_t are utilized. The communication range d_c of the sensors are at least twice the sensing range, $d_c \geqslant 2d_t$ [2].

The sensing-neighboring degree, w is defined as the number of sensors within sensing range d_t of each other. Given d_{ij}, the distance between sensors i and j, then define adjacency as $a_{ij} = 1$, if $d_{ij} \leqslant d_t$ and $a_{ij} = 0$, otherwise. Then, the sensing-neighboring degree of i^{th} sensor is $w_i = \sum_{i \neq j} a_{ij}$. The communication-neighboring degree, v is defined as the number of sensors within communication range d_c of each other. Define adjacency $a_{ij} = 1$ if $d_{ij} \leqslant d_c$ and $a_{ij} = 0$, otherwise and the communication-neighboring degree of i^{th} sensor is $v_i = \sum_{i \neq j} a_{ij}$. Now, the problems can be stated as: (1) What is the probability of detecting a randomly located target by at least one sensor given that N sensors are deployed? The solution to this problem can be considered as a deployment quality measure (DQM). (2) What is required number of sensors to provide the required deployment quality measure p_t? (3) What are the average sensing- and communication-neighboring degrees given that the communication range is at least twice the sensing range $d_c \geqslant 2d_t$.

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II. Analytical Model

Assume that the positions of a set of sensors be uniform randomly distributed in a rectangular field where the length and the width are D_1 and D_2, respectively, where $D_1 \leqslant D_2$. The distance between two random points d in a rectangular field is defined as the random variable $\mathbf{D}$. Then the probability density function and the cumulative distribution functions are defined in Equations 1 and 2 [3], respectively.

$$f_D(\xi D_1) = \frac{1}{D_1} \begin{cases} \begin{array}{r} 2\zeta^2\xi^3 + 2\zeta\xi\pi, \\ -4\zeta\xi^2(1+\zeta), \end{array} & 0 \leqslant \xi < 1, \\ \begin{array}{r} 4\zeta\xi\sqrt{\xi^2-1} \\ -2\zeta\xi(2\xi+\zeta) \\ +4\zeta\xi\sin^{-1}(1/\xi), \end{array} & 1 \leqslant \xi < \zeta^{-1}, \\ \begin{array}{r} 4\zeta\xi\sqrt{\xi^2-1} \\ +4\zeta^2\xi\sqrt{\xi^2-\zeta^{-2}} \\ -2\xi(\zeta^2\xi^2+1+\zeta^2) \\ +4\zeta\xi\sin^{-1}(1/\xi) \\ -4\zeta\xi\cos^{-1}(1/(\zeta\xi)), \end{array} & \zeta^{-1} \leqslant \xi < \sqrt{1+\zeta^{-2}}, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$$F_D(\xi D_1) = \begin{cases} 0, & \xi < 0 \\ \zeta\xi^2(\frac{1}{2}\zeta\xi^2 - \frac{4}{3}\xi(1+\zeta) + \pi, & 0 \leqslant \xi < 1, \\ \frac{2}{3}\zeta\sqrt{\xi^2-1}(2\xi^2+1) & \\ -\frac{1}{6}\zeta(8\xi^3 + 6\zeta\xi^2 - \zeta) & \\ +2\zeta\xi^2\sin^{-1}(1/\xi), & 1 \leqslant \xi < \zeta^{-1} \\ \frac{2}{3}\zeta\sqrt{\xi^2-1}(2\xi^2+1) & \\ -\frac{1}{2}\zeta^2(\xi^4 + 2\xi^2 - \frac{1}{3}) & \\ +\frac{2}{3}\sqrt{\xi^2-\zeta^{-2}}(2\zeta^2\xi^3+1) & \\ +\frac{1}{6}\zeta^{-2} - \xi^2 & \\ +2\zeta\xi^2\sin^{-1}1/\xi & \\ -2\zeta\xi^2\cos^{-1}1/\zeta\xi & \zeta^{-1} \leqslant \xi < \sqrt{1+\zeta^{-2}} \\ 1, & \sqrt{1+\zeta^{-2}} \leqslant \xi \end{cases} \tag{2}$$

where $\zeta = D_1/D_2 \leqslant 1$ is the shape parameter and $\xi = d/D_1$. Binary detector can be formulated as

$$g_D(x) = \begin{cases} 1, & x \leqslant d_t \\ 0, & x > d_t \end{cases} \tag{3}$$

where $x \geqslant 0$ is the sensor-to-target distance, d_t is the sensing range and $g(x)$ is the detection probability function defined with the random variable $x \in \mathbf{D}$. Assume that a target and a binary detector are positioned randomly in a rectangle region, then the expected value of target detection probability is

$$E\{g_D(\mathbf{x})\} = \int_0^\infty g_D(\mathbf{x}) f_D(x) dx = \int_0^{d_t} f_D(x) dx \tag{4}$$

By definition, this is the value of cumulative distribution function for d_t defined in Equation 2. That is, $E\{g_D(\mathbf{x})\} = F_D(d_t)$. We can define a Bernoulli trial as: Chose two random positions in the rectangle, assume that the target is located on one of the points and there is a sensor on the other point. If the distance between these two random points is smaller than d_t, the target is detected successfully and the trial fails otherwise. The expected value of this Bernoulli trial is defined in Equation 4. By definition, the expected value of a Bernoulli trial is equal to the success probability. Hence, the detection probability is $p = F_D(d_t)$. Repeating Bernoulli trials N times produces binomial distribution. If N sensors are deployed, then the probability that a randomly positioned target is detected by k of the sensors follows this binomial distribution. Hence,

$$p(N,k) = \binom{N}{k} p^k (1-p)^{N-k} = \binom{N}{k} F_D(d_t)^k (1-F_D(d_t))^{N-k} \tag{5}$$

Equation 5 cannot be approximated by a Poisson distribution because $\lim_{N\to\infty} NF_D(d_t) \to \infty$. However, since $NF_D(d_t)$ is large enough, Equation 5 can be approximated by normal distribution $\mathbb{N}(\mu, \sigma)$ where $\mu = NF_D(d_t)$ and $\sigma = NF_D(d_t)(1 - F_D(d_t)$. The probability that the target is detected by at least one sensor is

$$p_d = 1 - p(N,0) = 1 - (1 - F_D(d_t))^N. \tag{6}$$

where p_d is the solution of question one. Equation 6 can be expressed as the complement of the probability that none of the sensors detect the target. Using Equation 6, one can derive N for a predetermined target detection probability value p_t, then the required number of sensors for a given DQM level p_t is

$$N = \left\lceil \frac{\log(1-p_t)}{\log(1-F_D(d_t))} \right\rceil. \tag{7}$$

Concentrating on any randomly deployed sensor, we can define a Bernoulli trial as whether any other randomly deployed sensor is in the sensing range. If N sensors are deployed, then the sensing-neighboring degree of a sensor is the expected value of the binomial distribution obtained by $N-1$ Bernoulli trials. Consequently, the sensing-neighboring degree is

$$w = E\{p(N-1,k)\} = (N-1)F_D(d_t) \tag{8}$$

Since the communication range d_c is at least $2d_t$, then we can define the trial according to the communication range, hence the communication-neighboring degree is

$$v \geqslant (N-1)F_D(2d_t). \tag{9}$$

Since $F_D(2d_t) > F_D(d_t)$, if the network is designed according to the communication range, breach holes will exist in the sensing coverage. In the next section, we present numerical evaluation of the proposed analytical deployment quality measure.

III. Numerical Evaluation

The simulations are performed with Matlab. A set of sensor positions are determined randomly in a rectangle. For each run, a target is assumed to be located on a random position. If the distance between the target and any sensor is smaller than the sensing range, then the run is assumed to be a success, otherwise it is a failure. The results are averages of 1000 runs for each deployed sensor set. The effect of the number of sensors and the detector range on the DQM are shown in Fig. 1 and Fig. 2, respectively. The variances in the simulations are in the acceptable range and verify the correctness of the analytical model.

Cumulative distribution functions are monotonic increasing functions. Hence, as the sensing range increase, $F_D(d_t)$ increases and the probability of target detection p_d decreases (see Equation 6). This is depicted in Fig. 1; for $N = 10, 20, 30$, the deployment quality measure is plotted as the sensing range increases. As the number of deployed sensors increase, the provided DQM level increases (Fig. 2). Hence, for denser deployments, the required DQM is provided with sensors whose ranges are smaller. Notice, that when the sensing range is equal to the width of the field, the DQM is one and only a couple of randomly deployed sensors are enough to cover the field as seen in Fig. 3.

Keeping the field area constant as 3000 m^2, the shape of the field is influential on the required number of sensors as seen in Fig. 4, where the sensing range is 10 m. As the field gets more uniform in shape, fewer sensors are enough to provide the required DQM. When the shape parameter is small, more sensors are required. As the required deployment quality increases, more sensors are required. For example, when $\zeta = D_1/D_2 = 0.0083$, 58 sensors are required to provide a deployment quality of 0.85 target detection probability.

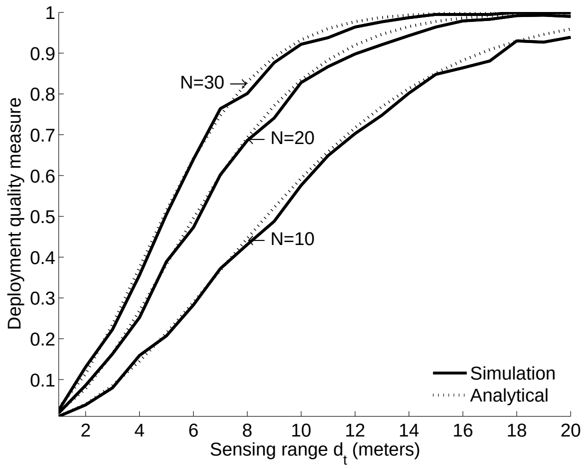

Fig. 1. The effect of sensing range on the DQM is verified with simulations where $D_1 = 30$ m., $D_2 = 100$ m. and $N = 10, 20, 30$.

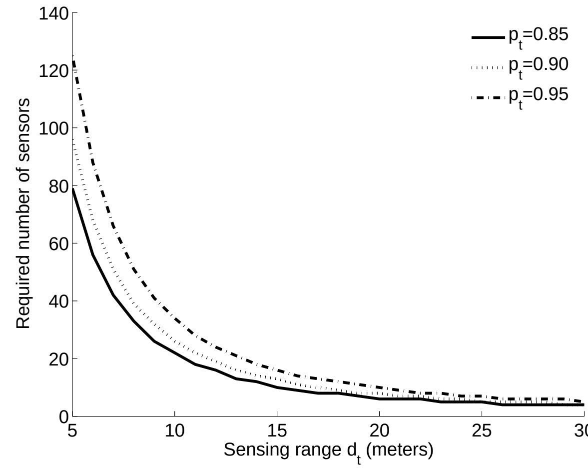

Fig. 3. Effect of sensing range on the required number of sensors where $D_1 = 30$ m., $D_2 = 100$ m. and $p_t = 0.85, 0.90, 0.95$.

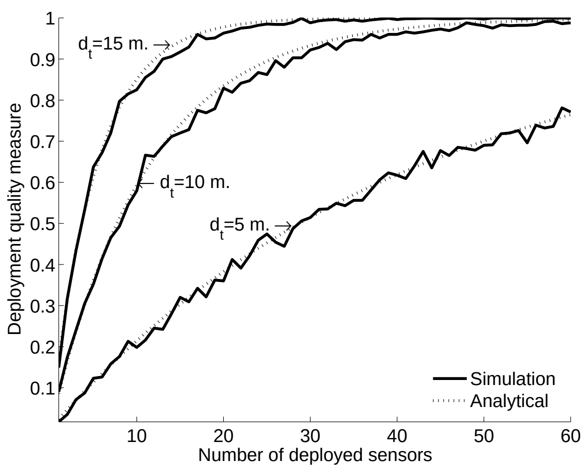

Fig. 2. The effect of number of sensors on the DQM is verified with simulations where $D_1 = 30$ m., $D_2 = 100$ m. and $d_t = 5, 10, 15$ m.

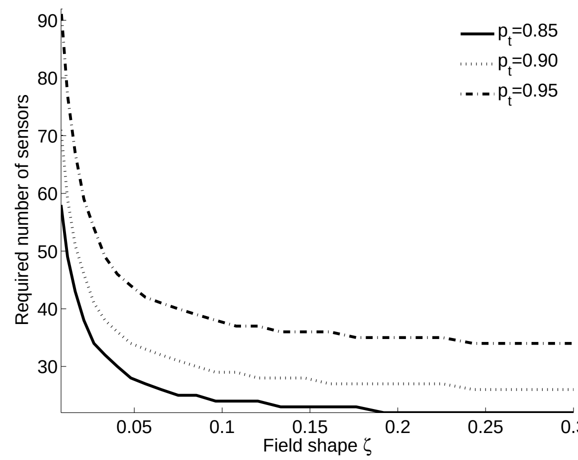

Fig. 4. Effect of field width $\zeta = D_1/D_2$ on the required number of sensors where total area is 3000 m^2, $d_t = 10$ m. and $p_t = 0.85, 0.90, 0.95$.

However, if $\zeta = 0.30$, deploying 22 sensors is adequate. The effect of the field shape is more influential when the shape parameter is quite small. That is, when the field is a rather narrow and long region, more sensors are required. When ζ is larger than 0.1, shape does not affect the required number of sensors much.

The average sensing- and communication-neighboring degrees increase as the sensing range increases. For example, when 10 sensors with 7 m. of sensing range is utilized, the average sensing neighboring degree is 1.02. The average communication degree is 3.15 if the communication range of the sensor is 14 m. When the network is planned according to the sensing-coverage, it can be considered as over-engineered in terms of the communication degree. Hence, high redundancy for communication is provided if the network is planned according to the sensing that is the main functionality of the network.

IV. Conclusions

An analytical model to determine the deployment quality is proposed in this paper where binary detectors are utilized. Using this model, it is possible to calculate the required number of sensors to provide the predetermined deployment quality level. Furthermore, some routing protocols depend on the neighboring degree of sensors. The sensing- and communication neighboring degrees can be calculated with this model. The analytical evaluation results closely match the simulation outcomes. The designer of the network may use the sensing- and communication-neighboring degrees as decision criteria along with the threshold DQM level.

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